# SYDNEY TECHNICAL HIGH SCHOOL

## HIGHER SCHOOL CERTIFICATE
## ASSESSMENT TASK 1

## DECEMBER 2012

# Mathematics

**General Instuctions**

- Working time - 70 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in questions 6 to 13
- Start each question on a new page

**Total marks - 53**

**Section 1 - 5 marks**

Attempt Questions 1 – 5.
Allow about 7 minutes for this section.

**Section 2 - 48 marks**

Attempt Questions 6 – 13.
Allow about 63 minutes for this section.

**Name :** _____

**Teacher :** _____

## Section 1

5 marks
Attempt Questions 1 – 5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 5.
Do not remove the multiple-choice answer sheet from your answer booklet.

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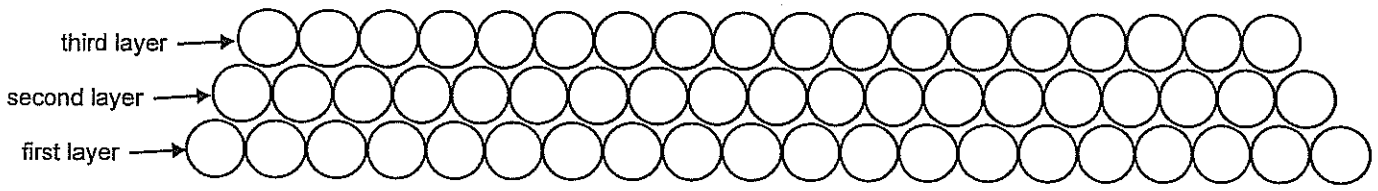
**1** If $x = -4$ is a root of the equation $2x^2 + kx + 4 = 0$, what is the value of $k$?

(A) 7
(B) 8
(C) 9
(D) 10

**2** In the first three layers of a stack of soup cans there are 20 cans in the first layer, 19 cans in the second layer and 18 cans in the third layer.




This pattern of stacking cans in layers continues.

The maximum number of cans that can be stacked in this way is

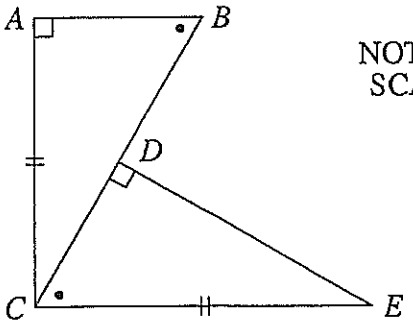
(A) 190
(B) 210
(C) 220
(D) 380

3

$\angle BAC = \angle CDE = 90°$

$AC=CE$

$\angle ABC= \angle DCE$




NOT TO SCALE

Consider these two statements:

I. $\Delta ABC \mid\mid\mid \Delta DCE$

II. $\Delta ABC \equiv \Delta DCE$

Which of the above statements are true ?

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

4 The coordinates of the focus of a parabola are ( 0 , 6 ) and the coordinates of its vertex are ( 0 , 4 ).

Which of the following could be the equation of the parabola ?

(A) $x^2 = 8y$

(B) $x^2 + 32 = 8y$

(C) $x^2 = 8(y - 6)$

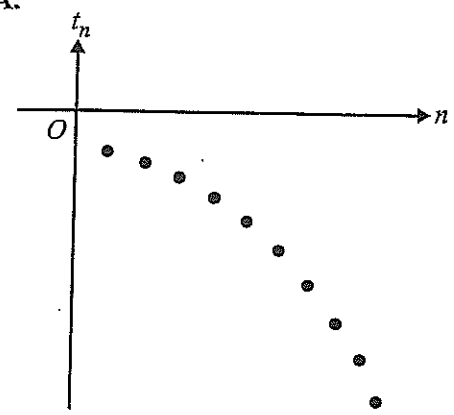
(D) $(x - 4)^2 = 8y$

5 The $n$th term in a geometric sequence is $t_n$ .

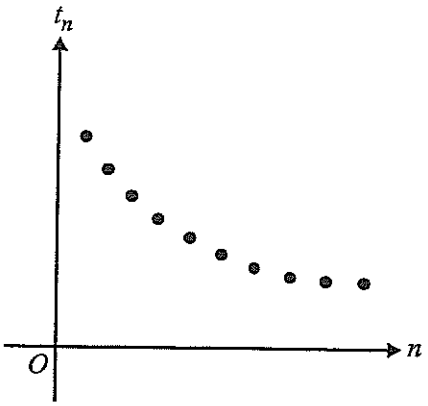
The common ratio is greater than one.

A graph that could be used to display the terms of this sequence is

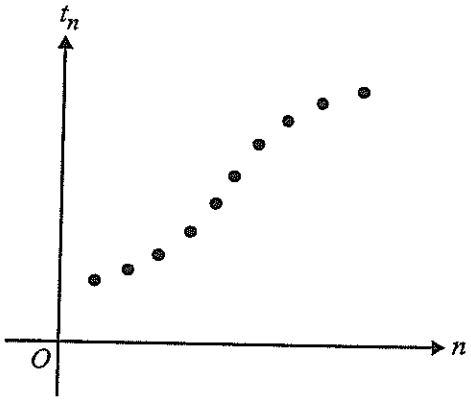
**A.**



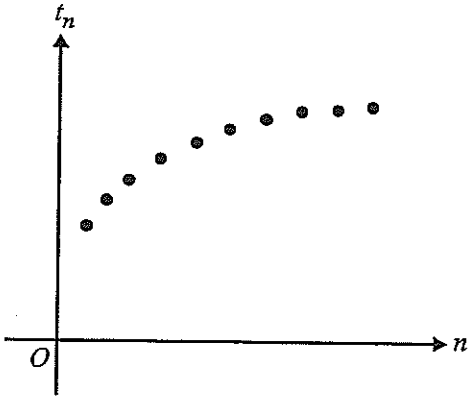

**B.**




**C.**




**D.**



## Section 2

48 marks
Attempt Questions 6 – 13
Allow about 63 minutes for this section
Start each question on a new page

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### Question 6 (6 marks)

a) For the series $2 + 13 + 24 + 35 + \ldots\ldots\ldots$

i) Find the $45^{th}$ term. 2

ii) Find the sum of the first 45 terms. 2

b) Find the values of $k$ for which the equation $5x^2 + 4x + k = 0$ has real roots. 2

### Question 7 (6 marks) Start a new page

a) Find the equation of the directrix of the parabola $x^2 = 12y$. 1

b) Find the sum of the first 16 terms of the series 2

$1 + 2^1 + 2^2 + 2^3 + \ldots\ldots\ldots\ldots$

c) Let $A$ and $B$ be the fixed points $(-3,4)$ and $(1,0)$ and let $P$ be the variable point $(x, y)$.

i) Write down an expression for the distance $PA$ in terms of $x$ and $y$. 1

ii) Find the locus of $P$ given that $P$ moves so that it is equidistant from $A$ and $B$. 2

**Question 8** (6 marks) Start a new page

a) If $\alpha$ and $\beta$ are the roots of the quadratic equation $3x^2 + 10x + 4 = 0$,

i) find the value of $\alpha\beta$ 1

ii) find the value of $\frac{2}{\alpha} + \frac{2}{\beta}$. 1

b) Find the value of $b$ and $c$ if 2

$686 \ , \ b \ , \ c \ , \ 2$

are 4 consecutive terms of a geometric sequence.

c) Find all real numbers $x$ which satisfy the equation 2

$$x^4 = 2(x^2 + 12)$$

**Question 9** (6 marks) Start a new page

a) The first three terms of a geometric sequence are 2

**12 , 18 , 27 , .....**

i) Find the common ratio of this sequence.

ii) Find the $16^{th}$ term of the sequence, giving your answer correct to the nearest whole number.

b) Find the equation of the tangent to the parabola $x^2 = 4y$ at the point $(2, 1)$. 2

c) The quadratic equation $x^2 + bx + c = 0$ has roots $5 - \sqrt{2}$ and $5 + \sqrt{2}$. 2

Find the values of $a$ and $b$.

**Question 10** (6 marks) Start a new page

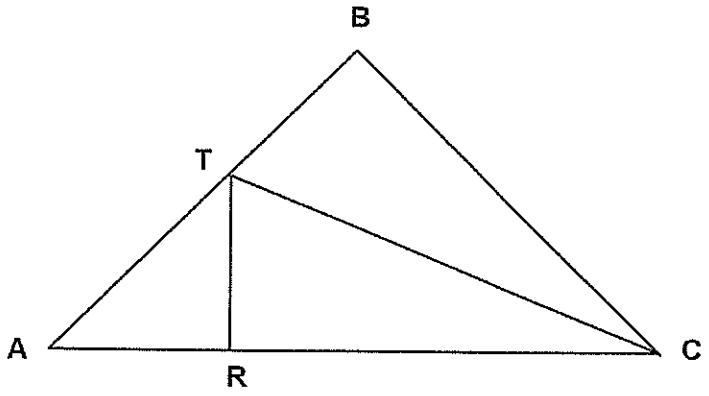
a) The sum of the first $n$ terms of a series is given by $S_n = 2n^2 + 7n$. 1

Find the first 3 terms of the series.

b) In the diagram below ABC is a right angled isosceles triangle with $\angle ABC = 90°$.

TC bisects $\angle ACB$.

TR is perpendicular to AC.




Copy or trace this diagram into your answer booklet.

i) Show that $\Delta BTC$ is congruent to $\Delta RTC$. 3

ii) Hence, show that $AR = TB$ 2

**Please turn over**

**Question 11** (6 marks) Start a new page

a) Evaluate $\sum_{n=3}^{5} \frac{(-1)^n}{n}$ 1

b) For what values of $k$ is the quadratic expression $x^2 - kx + k + 3$ 2

positive for all real values of $x$ ?

c) Consider the quadratic equation $3x^2 + (k+2)x + 8k = 0$. 3

Find the value of $k$ if the product of its roots is equal to twice the sum of its roots.

**Question 12** (6 marks) Start a new page

a) The equation of a parabola is $y = \frac{1}{4}(x^2 - 8x + 24)$

i) Find the coordinates of the vertex. 2

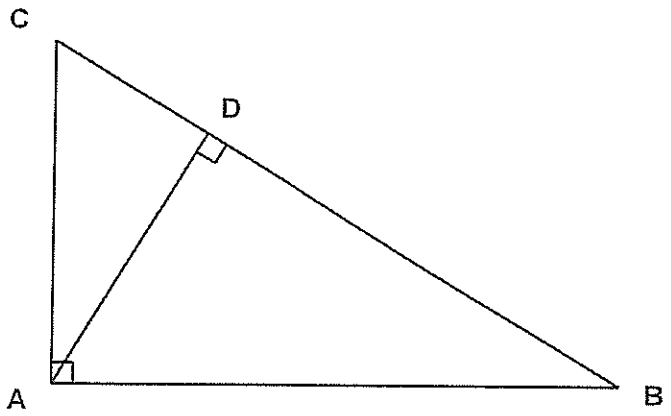
ii) Find the coordinates of the focus. 1

b) For what values of $m$ is the line $y = m(x+1)$ 3

a tangent to the parabola $y = 2x^2$ ?

**Please turn over**

**Question 13** (6 marks) Start a new page




Triangle ABC is right angled at A.

AD is perpendicular to BC.

Copy or trace this diagram into your answer booklet.

i) Show that $\Delta ABC$ is similar to $\Delta DBA$ 2

ii) Show that $(AB)^2 = BD \times BC$ 1

iii) A perpendicular from D to AB meets AB at E. 3

Show that $BE \times (BC)^2 = (AB)^3$.

**End of paper**

# SOLUTIONS

1. C
2. B
3. A
4. B
5. A

6. a) $a = 2 \quad d = 11$

i) $T_{45} = 2 + 44 \times 11$

$= 486$

ii) $S_{45} = \frac{45}{2}[4 + 44 \times 11]$

$= 10980$

b) $\Delta \geq 0$

$16 - 4 \times 5 \times k \geq 0$

$k \leq \frac{4}{5}$

7. a) $y = -3$

b) $a = 1 \quad r = 2$

$S_{16} = \frac{1(2^{16} - 1)}{2 - 1}$

$= 65535$

c)

i) $PA = \sqrt{(x+3)^2 + (y-4)^2}$

ii) $(x+3)^2 + (y-4)^2 = (x-1)^2 + y^2$

$x^2 + 6x + 9 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2$

$8x - 8y + 24 = 0$

$x - y + 3 = 0$

8. a) i) $\alpha\beta = \frac{4}{3}$

ii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2(\alpha + \beta)}{\alpha\beta}$

$= \frac{2 \times \frac{-10}{3}}{\frac{4}{3}}$

$= -5$

b) $a = 686$

$ar^3 = 2$

$\therefore r^3 = \frac{1}{343}$

$r = \frac{1}{7}$

$\therefore b = 98 \quad c = 14$

c) $x^4 - 2x^2 - 24 = 0$

let $u = x^2$

$u^2 - 2u - 24 = 0$

$(u - 6)(u + 4) = 0$

$u = 6, -4$

$\therefore x^2 = 6$ or $x^2 = -4$ no real solution

$\therefore x = \pm\sqrt{6}$

9. a) i) $r = \frac{18}{12}$

$= 1.5$

ii) $T_{16} = 12 \times 1.5^{15}$

$\doteq 6255$

b) $y = \frac{1}{4}x^2$

$y' = \frac{1}{2}x$

when $x = 2$

$m_t = 1$

$\therefore y - 1 = 1(x-2)$

$y = x - 1$

c) $\alpha + \beta = 5 - \sqrt{2} + 5 + \sqrt{2}$

$= 10$

$\alpha\beta = (5-\sqrt{2})(5+\sqrt{2})$

$= 23$

$\therefore x^2 - 10x + 23 = 0$

$\therefore a = -10, \quad b = 23$

10. a) $S_n = 2n^2 + 7n$

$S_1 = 9 \quad \therefore T_1 = 9$

$S_2 = 22 \quad \therefore T_2 = 13$

$S_3 = 39 \quad \therefore T_3 = 17$

$\therefore 9, 13, 17$

b) i) In $\triangle BTC$ and $\triangle RTC$

TC is common

$\angle RCT = \angle BCT$ (given TC bisects $\angle ABC$)

$\angle TRC = \angle TBC = 90°$ (given)

$\therefore \triangle BTC \equiv \triangle RTC$ (AAS)

ii) $TB = TR$ (corresponding sides in congruent triangles)

$\angle TAR = 45°$ ($\triangle ABC$ is right angled isosceles)

$\therefore \angle ATR = 45°$ (angle sum of $\triangle ATR$)

$\therefore AR = TR$ (sides opposite equal angles equal)

$\therefore AR = TB$ (both equal TR)

11 a) $\sum_{n=3}^{5} \frac{(-1)^n}{n}$

$= -\frac{1}{3} + \frac{1}{4} - \frac{1}{5}$

$= -\frac{17}{60}$

b) positive definite $a > 0$ ✓

$\Delta < 0$

$(-k)^2 - 4(1)(k+3) < 0$

$k^2 - 4k - 12 < 0$

$(k-6)(k+2) < 0$

$-2 < k < 6$

c) $\alpha\beta = 2(\alpha + \beta)$

$\frac{8k}{3} = -2\left(\frac{k+2}{3}\right)$

$10k = -4$

$k = -\frac{2}{5}$

12 c) $y = \frac{1}{4}(x^2 - 8x + 24)$

$4y = x^2 - 8x + 24$

$4y - 8 = x^2 - 8x + 16$

$4(y-2) = (x-4)^2$

i) Vertex $(4, 2)$

ii) focus $(4, 3)$

b) when solve simultaneously there is 1 solution only.

$\Rightarrow \Delta = 0$

$\therefore 2x^2 = m(x+1)$

$2x^2 - mx - m = 0$

$\Delta = 0$

$\therefore (-m)^2 - 4(2)(-m) = 0$

$m^2 + 8m = 0$

$m(m+8) = 0$

$m = 0, -8$

13. i) In $\triangle ABC$ and $\triangle DBA$

$\angle B$ is common

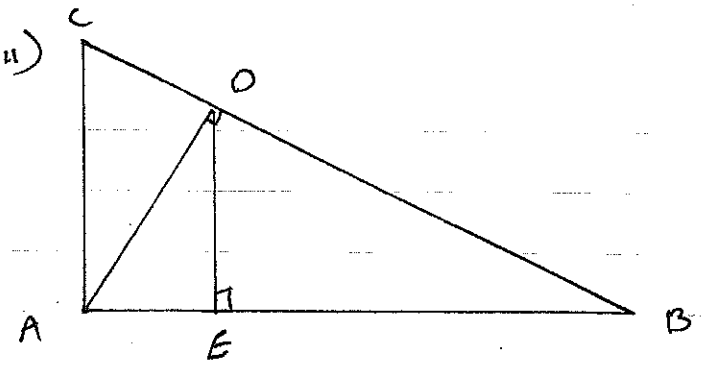
$\angle BAC = \angle ADB = 90^\circ$ (given)

$\therefore \triangle ABC \parallel\!\parallel \triangle DBA$ (equiangular)

ii) $\frac{AB}{DB} = \frac{BC}{AB}$ (corresponding sides of similar triangles)

$\therefore AB^2 = BD \times BC$

iii)




$\triangle ABC \parallel\!\parallel \triangle EBD$ (equiangular)

$\therefore \frac{AB}{EB} = \frac{BC}{BD}$ (corresponding sides of similar triangles)

$\therefore AB \times BD = BC \times BE$

$\therefore AD \times \frac{AB^2}{BC} = BC \times BE$ from part ii)

$\therefore AB^3 = BC^2 \times BE$